

2.1 Fractions and Estimation

Essential Question How can you use estimation to check that your answer is reasonable?

1 ACTIVITY: Using Models for Fractions

Work with a partner. Use the model for the whole to draw a model for the given fractions.

Whole	Model for the Whole	Fractions	Model for Fraction
a. Sample: Circle		$\frac{5}{8}$	
b. Circle		$\frac{3}{4}, \frac{5}{12}, \frac{4}{6}$	
c. Rectangle		$\frac{3}{5}, \frac{4}{5}, \frac{7}{10}$	
d. Counters		$\frac{1}{2}, \frac{3}{8}, \frac{3}{4}$	
e. Piece of paper		$\frac{7}{8}, \frac{1}{8}, \frac{1}{4}$	

2 ACTIVITY: Estimating Sums and Differences

Work with a partner. Add or subtract. Then check your answer by using one of the models in Activity 1 to estimate the sum or difference.

a. Sample:

$$\begin{aligned} \frac{1}{6} + \frac{1}{4} &= \frac{2}{12} + \frac{3}{12} \\ &= \frac{2+3}{12} \\ &= \frac{5}{12} \end{aligned}$$

Write with common denominator.

Add numerators.

Simplify.



Sum is less than one half.

b. $\frac{1}{3} + \frac{1}{4}$

c. $\frac{5}{8} + \frac{1}{3}$

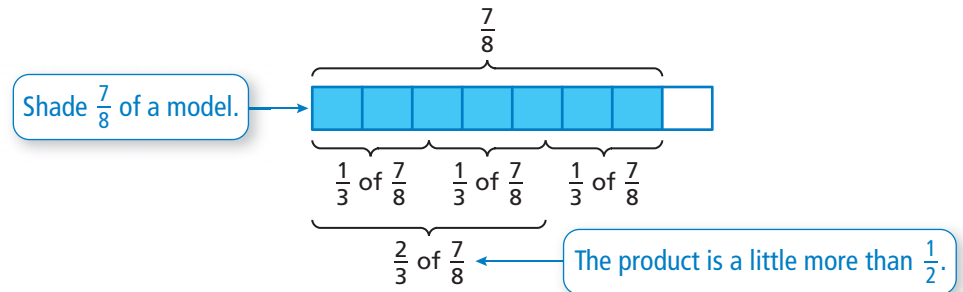
d. $\frac{7}{8} - \frac{1}{3}$

e. $\frac{2}{3} - \frac{4}{9}$

3 ACTIVITY: Estimating Products

Work with a partner. Use a fraction model to choose 0 , $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, or 1 as the best estimate of the product.

a. Sample: $\frac{2}{3} \times \frac{7}{8}$



∴ So, the best estimate is $\frac{1}{2}$.

b. $\frac{1}{5} \times \frac{3}{10}$

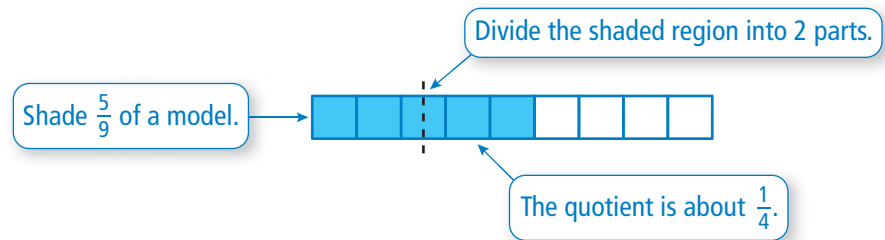
c. $\frac{3}{4} \times \frac{5}{7}$

d. $\frac{7}{8} \times \frac{7}{8}$

4 ACTIVITY: Estimating Quotients

Work with a partner. Use a fraction model to choose 0 , $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, or 1 as the best estimate of the quotient.

a. Sample: $\frac{5}{9} \div 2$



∴ So, the best estimate is $\frac{1}{4}$.

b. $\frac{3}{5} \div 3$

c. $\frac{1}{2} \div 8$

d. $\frac{5}{6} \div 2$

What Is Your Answer?

5. **IN YOUR OWN WORDS** How can you use estimation to check that your answer is reasonable? Give some examples.

Practice

Use what you learned about estimation to complete Exercises 7–14 on page 48.

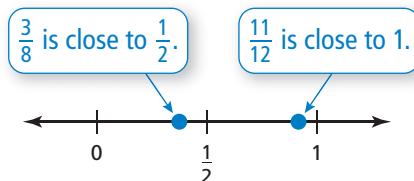
EXAMPLE 1 Estimating Products

Key Vocabulary

underestimate, p. 47
overestimate, p. 47
compatible numbers,
p. 47

Estimate the product by rounding to 0, $\frac{1}{2}$, or 1.

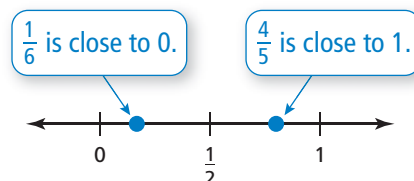
a. $\frac{3}{8} \times \frac{11}{12}$



$$\frac{3}{8} \times \frac{11}{12} \approx \frac{1}{2} \times 1 = \frac{1}{2}$$

∴ $\frac{3}{8} \times \frac{11}{12}$ is about $\frac{1}{2}$.

b. $\frac{4}{5} \times \frac{1}{6}$



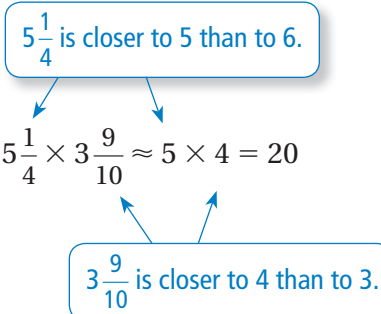
$$\frac{4}{5} \times \frac{1}{6} \approx 1 \times 0 = 0$$

∴ $\frac{4}{5} \times \frac{1}{6}$ is about 0.

EXAMPLE 2 Estimating with Mixed Numbers

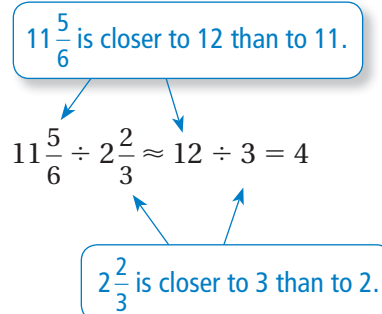
Estimate the product or quotient by rounding each mixed number to the nearest whole number.

a. $5\frac{1}{4} \times 3\frac{9}{10}$



∴ $5\frac{1}{4} \times 3\frac{9}{10}$ is about 20.

b. $11\frac{5}{6} \div 2\frac{2}{3}$



∴ $11\frac{5}{6} \div 2\frac{2}{3}$ is about 4.

Reading

The symbol \approx means "approximately equal to."

On Your Own

Estimate the product or quotient.

1. $\frac{1}{9} \times \frac{4}{5}$

2. $\frac{9}{10} \times \frac{5}{12}$

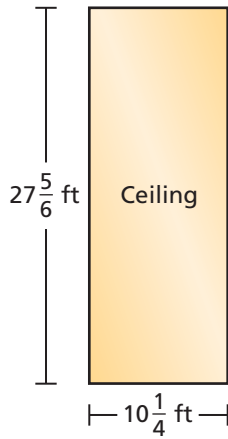
3. $2\frac{7}{8} \times 6\frac{1}{3}$

4. $24\frac{1}{5} \div 3\frac{1}{2}$

Now You're Ready
Exercises 7–22

An **underestimate** is an estimate that is less than the exact answer while an **overestimate** is greater than the exact answer.

EXAMPLE 3 Using an Overestimate



One gallon of paint covers 350 square feet. Is 1 gallon of paint enough to cover the rectangular ceiling? Explain.

$$\begin{aligned}
 A &= \ell w && \text{Write the formula for the area of a rectangle.} \\
 &= 27\frac{5}{6} \times 10\frac{1}{4} && \text{Substitute for } \ell \text{ and } w. \\
 &\approx 28 \times 11 && \text{Round } 27\frac{5}{6} \text{ up to 28. Round } 10\frac{1}{4} \text{ up to 11.} \\
 &= 308 && \text{Multiply.}
 \end{aligned}$$

Because 308 is an overestimate and is less than 350, 1 gallon of paint is enough.

On Your Own

5. In Example 3, a hallway wall is $9\frac{3}{4}$ feet by $64\frac{1}{3}$ feet. Are 2 gallons of paint enough to cover the wall? Explain.

Compatible numbers are numbers that are easy to compute mentally.

EXAMPLE 4 Using a Compatible Number

Reading

The term "range" refers to the region where a Florida panther lives.



The range of a male Florida panther is about $3\frac{3}{4}$ times the range of a female Florida panther. The range of a male is about 275 square miles. Estimate the range of a female Florida panther.

$$\begin{aligned}
 275 \div 3\frac{3}{4} &\approx 275 \div 4 && \text{Round } 3\frac{3}{4} \text{ to the nearest whole number, 4.} \\
 &\approx 280 \div 4 && \text{280 is close to 275 and is divisible by 4.} \\
 &= 70 && \text{Divide.}
 \end{aligned}$$

275 is not evenly divisible by 4.

So, the range of a female Florida panther is about 70 square miles.

On Your Own

6. There are about 100 Florida panthers in South Florida. A scientist wants $\frac{3}{8}$ of the panthers fitted with tracking collars. Estimate the number of panthers to be fitted with collars.

Now You're Ready
Exercises 24–31

2.1 Exercises

Vocabulary and Concept Check

Tell whether you would use *rounding* or *compatible numbers* to estimate the product or quotient. Explain your reasoning.

1. $2\frac{1}{6} \times 5\frac{11}{12}$

2. $7\frac{3}{4} \div 1\frac{7}{9}$

3. $\frac{2}{5} \times \frac{7}{8}$

4. $34 \div 8\frac{2}{3}$

5. Copy and complete the table to estimate the quotient $77 \div 4\frac{2}{5}$.

How to Round	Estimate
Round 77 to the nearest hundred.	<input type="text"/> $\div 4 =$ <input type="text"/>
Round 77 to the nearest ten.	<input type="text"/> $\div 4 =$ <input type="text"/>
Round 77 to the nearest compatible number.	<input type="text"/> $\div 4 =$ <input type="text"/>

6. **NUMBER SENSE** In Exercise 5, the quotient $77 \div 4\frac{2}{5}$ equals $17\frac{1}{2}$. What do you notice about the estimates in the table?

Practice and Problem Solving

Estimate the product or quotient.

1 2 7. $\frac{4}{7} \times \frac{1}{6}$

8. $\frac{9}{10} \times \frac{5}{9}$

9. $\frac{1}{5} \times \frac{7}{8}$

10. $\frac{8}{15} \times \frac{5}{6}$

11. $\frac{3}{4} \times \frac{1}{3}$

12. $\frac{2}{3} \times \frac{1}{7}$

13. $\frac{11}{8} \div 3$

14. $\frac{7}{9} \div 2$

15. $\frac{5}{13} \times \frac{4}{5}$

16. $\frac{1}{10} \times \frac{3}{16}$

17. $\frac{5}{6} \times \frac{7}{12}$

18. $\frac{3}{4} \times \frac{7}{9}$


19. $8\frac{3}{4} \times 2\frac{1}{2}$

20. $14\frac{11}{15} \times 4\frac{3}{7}$

21. $42\frac{2}{9} \div 6\frac{6}{7}$

22. $19\frac{1}{2} \div 4\frac{7}{8}$

23. **ERROR ANALYSIS** Describe and correct the error in estimating the product.

 $\frac{5}{12} \times \frac{9}{10} \approx 0 \times 1 = 0$

Use compatible numbers to estimate the product or quotient.

4 24. $61 \div 4\frac{3}{8}$

25. $48 \div 6\frac{7}{12}$

26. $151 \times \frac{2}{5}$

27. $203 \times \frac{6}{7}$

28. $152 \div 6\frac{3}{11}$

29. $135 \div 19\frac{7}{10}$

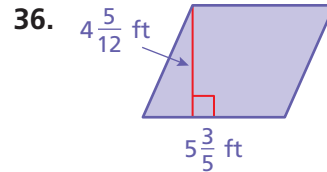
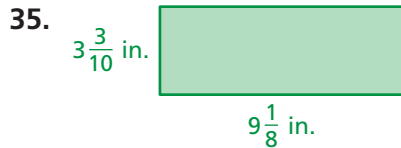
30. $155 \div 7\frac{2}{9}$

31. $177 \div 8\frac{5}{6}$

32. **FLOWERS** You plant 25 flower bulbs in a garden. About $\frac{3}{4}$ of the flowers bloom. Estimate the number of flowers that bloom.

33. **RACECAR** The height of a racecar is $46\frac{7}{8}$ inches. A model of the racecar is $2\frac{7}{9}$ inches tall. About how many times greater is the height of the racecar than the height of the model?
34. **BREAD** A recipe for a loaf of bread calls for $3\frac{1}{4}$ cups of flour. About how many loaves of bread can you make with 25 cups of flour?

GEOMETRY Estimate the area of the rectangle or parallelogram. Did you overestimate or underestimate the area? Explain.



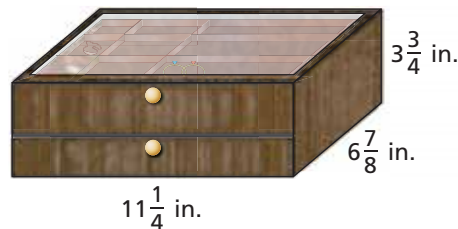
Estimate the value of the expression.

37. $6\frac{1}{4} \times 9\frac{3}{7} \div 2\frac{7}{8}$

38. $11\frac{2}{3} \div 3\frac{7}{12} \times 6\frac{2}{5}$

39. $100\frac{3}{8} \div \left(3\frac{5}{6} \times 5\frac{2}{9}\right)$

40. **WALLPAPER** You cover a wall that is $8\frac{5}{8}$ feet by $17\frac{1}{4}$ feet with wallpaper. One roll of wallpaper covers 60 square feet. Are 3 rolls of wallpaper enough to cover the wall? Explain.
41. **Geometry** Find a low estimate and a high estimate for the surface area of the jewelry box. Explain how you found your answers.



Fair Game Review

what you learned in previous grades & lessons

Evaluate the expression.

42. $\frac{2 \times 18}{3}$

43. $\frac{4 \times 45}{5}$

44. $\frac{5 \times 14}{6}$

45. $\frac{3 \times 12}{8}$

46. **MULTIPLE CHOICE** Which expression does *not* need the Commutative Property of Addition or the Commutative Property of Multiplication to simplify?

(A) $18 + (x + 3)$

(B) $6(9x)$

(C) $(4 \cdot x) \cdot 11$

(D) $5 + 10x + 7$